Highly nondegenerate all-resonant optical parametric oscillator

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We show that a nondegenerate multi-frequency parametric oscillator has different properties compared with an usual three-wave parametric oscillator. We consider, as an example, a scheme of a resonant cw monolithic microwave-optical parametric oscillator based on high-Q whispering gallery modes excited in a nonlinear dielectric cavity. Such an oscillator may have an extremely low threshold and stable operation, and may be used in spectroscopy and metrology. The oscillator mimics devices based on resonant $\chi^{(3)}$ nonlinearity and can be utilized for efficient four-wave mixing and optical comb generation. Moreover, the oscillator properties are important for better understanding stability conditions of long-base interferometers with movable mirrors that are currently used for gravity wave detection.

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I. INTRODUCTION

Optical parametric oscillators (OPO) have been extensively studied since the discovery of lasers [1–3]. Properties of OPO are quite understood by now [4–6]. The cw-OPO have been considered as an ideal device that are able to generate at broad range of wavelengths. Because of their reliability and excellent stability cw-OPO are widely used, for example, in frequency chains [7, 8], optical frequency comb generators [9–12], and for preparation of nonclassical states of light [13].

One of the main restrictions of efficient parametric oscillations results from phase matching conditions. Index of refraction in nonlinear materials strongly depends on the frequency. This dependence brakes momentum conservation for pump and generated field photons propagating in a bulk material. To fulfill the phase matching conditions periodically poled materials are usually used [4].

Coupling of fields with significantly different frequencies, for example, a microwave field and light, is important practical as well as fundamental problem. Parametric processes may be helpful here. However, realization of phase matching for strongly nondegenerate parametric interactions is especially complicated. For example, index of refraction of $LiNbO_3$ differs more than twice for light and microwave fields. A way of solution of the problem was recently proposed and realized for a planar geometry [10] and for whispering gallery modes [14–20]. Efficient resonant interaction of optical whispering gallery modes and a microwave mode were achieved by engineering shape of a microwave resonator coupled to a dielectric optical cavity. To achieve interaction the optical cavity and the microwave resonator were pumped from outside. The outgoing light was modulated as the result of the interaction.

We here show that parametric interaction among waves with substantially different frequencies may significantly differ from usual OPO behavior. We theoretically study two closely related examples: a nondegenerate OPO that converts light into light and microwaves and an optical parametric process that converts light into light and mirror motion in long-base interferometers with moving mirrors [21, 22].

Both examples involve an optical cavity that has a large number of nearly equidistant modes. Each pair of these modes may interact via the microwave field. As a result, generation of two light modes with frequency difference equal to twice frequency of the microwave field is possible. This process resembles four-wave mixing processes in $\chi^{(3)}$ media [6], where Stokes and anti-Stokes fields are generated from a single pumping coherent field [23].

We show that the system also may be used for generation of a comb of harmonics if two-frequency optical pumping field is used. Unlike to usual comb generators based on parametric interaction [9–12], our system does not need application of a microwave field. Generation of harmonics occurs in the way similar to harmonic generation in resonant $\chi^{(3)}$ media [24, 25] and stimulated Raman scattering in droplets with $\chi^{(3)}$ nonlinearity [26–29]. Two-frequency pumping field leads to generation of the microwave field. The microwave field interacts with the pumping and creates equally spaced harmonics. The process is the most efficient when i) the frequency difference for the two-frequency pumping light corresponds to the resonant microwave frequency, and ii) the spectrum of optical modes is equidistant. Both these conditions may be fulfilled.

The optical properties of the strongly nondegenerate OPO are similar to the properties of usual atomic or molecular resonant structure. Really, our parametric system has narrow and stable resonances and results in four-wave mixing process as well as generation of multiharmonic field. Hence, we might say that we propose an

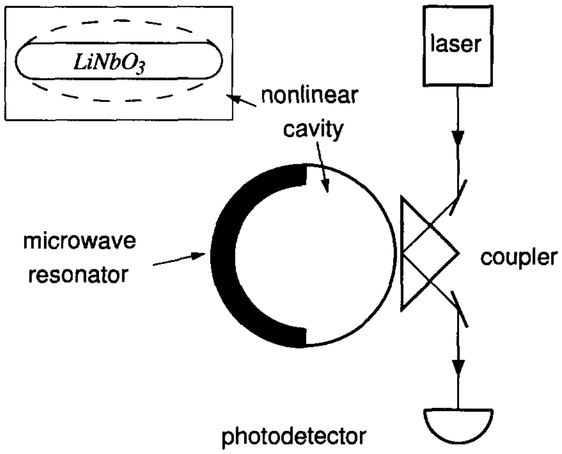

FIG. 1: Optical-microwave parametric oscillator. Insert: side view of the optical dielectric cavity. The boundaries of the cavity coincide with the boundaries of a spheroid, shown by dashed line.

"artificial atomic structure". However, our system is not "nonlinear enough", it has no spontaneous emission, and, therefore, it has many differences from a "natural atom". Some of these similarities and differences are discussed in the paper.

Our theoretical analysis is rather general. As an example of its application to a realistic system we consider OPO based on high-Q whispering gallery modes and propose a new configuration of a solid state monolithic OPO which converts light into light and microwaves. We suggest to design shape of the microwave resonator as in [16] and show that light modulation may appear without microwave pumping. The microwave field is generated from vacuum as the result of the parametric interaction.

We assume that pump laser radiation is sent into z-cut $LiNbO_3$ spheroid optical cavity via coupling diamond prism (see in Fig.1). Oblate spheroid cavity shape is essential to obtain a large free spectral range [15, 30]. The optical cavity is placed between two plates of microwave resonator. The resonant frequency of the microwave field can be adjusted to fit the frequency difference between optical modes by change of the resonator shape. Spectrum of the dielectric cavity may be engineered by changing profile of the index of refraction of the cavity material as well as shape of the cavity [31]. Due to $\chi^{(2)}$ nonlinearity of $LiNbO_3$ the modes of the microwave resonator and optical cavity are effectively coupled. This coupling increase significantly for resonant tuning of the fields due to high quality factors of the modes of optical cavity and microwave resonator, and small mode volumes [32–35].

We show that the threshold of oscillations is as low as μW of light pump power for realistic parameters. The stability of the signal may be better that of the pump due to high quality factor of the whispering gallery modes. Therefore, this OPO configuration not only gives a promise for use as a new configuration of optical microwave modulator, but also for use as a light source for optical frequency measurement and high precision spectroscopy.

The problem of strongly nondegenerate parametric instability also arises in a long-base interferometers with suspended movable mirrors. The nonlinearity has a ponderomotive origin there. Such interferometers are currently used for detection of gravity waves [21, 22]. In contrast to the mentioned above case of OPO a comparably low threshold of the oscillations is a disadvantage because it substantially reduces the detection sensitivity. We show that for an equidistant spectrum of the interferometer the threshold may be significantly increased. Ideally, if all parasitic modes are suppressed the interferometer might be stable even if the frequency of the oscillation of a mirror coincides with the free spectral range of the optical cavity.

The paper is organized as follows: In Sec. II we recall main properties of usual three-mode OPO. In Sec. III we discuss four-mode OPO. In Sec. IV we study conditions of generation of a frequency comb in all-resonant OPO with two-frequency optical pumping. In Sec. V we discuss stability of a cavity with moving mirrors.

II. DOWN-CONVERSION OF LIGHT INTO LIGHT AND MICROWAVES

Let us consider a nonlinear interaction of a coherent laser radiation, microwave field and generated light radiation (pump, idler, and signal respectively). Pump and signal electromagnetic waves are nearly resonant with different modes of an optical nonlinear dielectric cavity while microwave field is nearly resonant with a mode of the microwave resonator coupled to the dielectric cavity. We assume that only two modes of light and single mode of microwaves obeys to the resonant condition.

The Hamiltonian describing this system is (see, for example [36])

$$\hat{H} = \hbar\omega_a \hat{a}^\dagger \hat{a} + \hbar\omega_b \hat{b}^\dagger \hat{b} + \hbar\omega_c \hat{c}^\dagger \hat{c} + \hbar g(\hat{b}^\dagger \hat{c}^\dagger \hat{a} + \hat{a}^\dagger \hat{b}\hat{c}), \quad (1)$$

where ω_a and ω_b are the eigenfrequencies of the optical cavity modes, ω_c is the eigenfrequency of the microwave resonator mode, $\hat{a}$, $\hat{b}$, and $\hat{c}$ are the annihilation operators for these modes respectively,

$$g = \frac{\omega_a}{2}\chi^{(2)}\sqrt{\frac{4\pi\hbar\omega_c}{\mathcal{V}_c}}\left[\frac{1}{\mathcal{V}}\int_{\mathcal{V}} d\mathcal{V}\Psi_a\Psi_b\Psi_c\right] \quad (2)$$

is a coupling constant, $\chi^{(2)}$ is the electro-optic constant for the material of the dielectric cavity, $\mathcal{V}$ is the whispering gallery mode volume, $\mathcal{V}_c$ is the volume of the microwave field, Ψ_a, Ψ_b, and Ψ_c are the normalized dimensionless spatial distributions of the modes.

Using Hamiltonian (1) we derive equations of motion for the field operators

$$\dot{\hat{a}} = -i\omega_a\hat{a} - ig\hat{b}\hat{c}, \quad (3)$$

$$\dot{\hat{b}} = -i\omega_b\hat{b} - ig\hat{c}^\dagger\hat{a}, \tag{4}$$

$$\dot{\hat{c}} = -i\omega_c\hat{c} - ig\hat{b}^\dagger\hat{a}. \tag{5}$$

We consider an open system. To describe such systems appropriate decay and pumping terms should be introduced into Eqs. (3-5). The decay with necessity leads to leakage of quantum fluctuations into the system. To describe these fluctuations we use Langevin approach [3].

Let us introduce slow varying amplitudes for the mode operators as follows

$$\hat{a} = Ae^{-i\omega_0 t}, \hat{b} = Be^{-i\omega_- t}, \hat{c} = Ce^{-i\omega_M t}, \tag{6}$$

where ω_0 is the carrier frequency of the external pump of the mode $\hat{a}$, ω_- and ω_M are the frequencies of generated light and microwaves respectively. These frequencies are determined by the oscillation process and can not be controlled from outside. However, there is a ratio between them

$$\omega_0 = \omega_- + \omega_M. \tag{7}$$

Equations for the slow amplitudes of the intracavity fields follows from (3-5):

$$\dot{A} = -\Gamma_A A - igBC + F_A, \tag{8}$$

$$\dot{B} = -\Gamma_B B - igC^\dagger A + F_B, \tag{9}$$

$$\dot{C} = -\Gamma_C C - igB^\dagger A + F_M, \tag{10}$$

where

$$\Gamma_A = i(\omega_a - \omega_0) + \gamma,$$
$$\Gamma_B = i(\omega_b - \omega_-) + \gamma,$$
$$\Gamma_C = i(\omega_c - \omega_M) + \gamma_M,$$

F_A, F_B and F_C are the Langevin forces, γ and γ_M are optical and microwave decay rates respectively.

The Langevin forces are described by the following nonvanishing commutation relations:

$$\left[F_A(t)F_A^\dagger(t')\right] = \left[F_B(t)F_B^\dagger(t')\right] = 2\gamma\delta(t-t'), \tag{11}$$
$$\left[F_C(t)F_C^\dagger(t')\right] = 2\gamma_M\delta(t-t'),$$

and average values

$$\langle F_A\rangle = \sqrt{\frac{2\gamma W_A}{\hbar\omega_a}}, \quad \langle F_B\rangle = \langle F_C\rangle = 0, \tag{12}$$

where W_A is the power of the pumping of the mode from the outside. We assume that the fluctuations entering each mode from outside are in coherent state and are uncorrelated with each other.

Let us solve set (8-10) keeping expectation values only. Neglecting by optical saturation of the microwave oscillations we obtain from (8) and (10) in the steady state

$$\langle A\rangle = -i\frac{g}{\Gamma_A}\langle B\rangle\langle C\rangle + \frac{\langle F_A\rangle}{\Gamma_A}, \tag{13}$$

$$\langle C\rangle = -i\frac{g}{\Gamma_C}\langle B^*\rangle\langle A\rangle. \tag{14}$$

Substituting (13) and (14) into (9) we get

$$\langle\dot{B}\rangle + \langle B\rangle\left(\Gamma_B - \frac{g^2}{\Gamma_C^*}\frac{|\langle F_A\rangle|^2}{|\Gamma_A|^2}\left|1 + \frac{g^2|\langle B\rangle|^2}{\Gamma_A\Gamma_C}\right|^{-2}\right) = 0 \tag{15}$$

This equation has a nontrivial steady state solution if the expression in parentheses is equal to zero. From the real and imaginary parts of this expression we derive equations for the amplitude and frequency of the generated field

$$\left|1 + \frac{g^2|\langle B\rangle|^2}{\Gamma_A\Gamma_C}\right|^2 = \frac{\gamma_M}{\gamma}\frac{g^2}{|\Gamma_C|^2}\frac{|\langle F_A\rangle|^2}{|\Gamma_A|^2}, \tag{16}$$

$$\frac{\omega_b - \omega_-}{\gamma} = \frac{\omega_c - \omega_M}{\gamma_M}. \tag{17}$$

Expression for the oscillation threshold can be found using assumption that the right hand side term in (16) should exceed unity. Assuming the resonant tunings of all the fields $\Gamma_A = \Gamma_B = \gamma$, $\Gamma_C = \gamma_M$, introducing quality factors as $Q = \omega_0/(2\gamma)$ and $Q_M = \omega_M/(2\gamma_M)$, recalling $|F_A|^2/\gamma^2 = 4WQ/(\hbar\omega_0^2)$, and using expression (2) we derive expression for the threshold value for the pump power

$$W_{th} = \left(\frac{1}{\chi^{(2)}}\right)^2 \frac{\mathcal{V}_c}{\pi Q_M}\frac{\omega_0}{4Q^2}, \tag{18}$$

where we assumed that the normalized overlapping integral among modes is equal to 1/2.

Let us estimate the threshold power. For realistic parameters for a dielectric whispering gallery mode cavity coupled to a microwave resonator [16] $Q = 3\times 10^7$, $Q_M = 10^3$, and $\mathcal{V}_c = 10^{-7}$ cm^3 we get $W_{th} = 1$ μW.

Using (7) we rewrite (17) as

$$\omega_- = \frac{\omega_b + \dfrac{\gamma}{\gamma_M}(\omega_0 - \omega_c)}{1 + \dfrac{\gamma}{\gamma_M}}. \tag{19}$$

There is also a ratio between signal and idler amplitudes for the case when oscillations occur

$$\frac{|\langle B\rangle|^2}{|\langle C\rangle|^2} = \frac{\gamma_M}{\gamma} \tag{20}$$

Therefore, if $\gamma_M \gg \gamma$, the oscillation frequency is pulled to the center of the corresponding optical cavity resonance and photon number in the optical cavity exceeds the photon number in the microwave resonator. Otherwise the microwave frequency is pulled to the center of the microwave resonance.

Let us calculate now phase diffusion in the system. We represent the field operators in form

$$A = (|\langle A\rangle| + \delta A)e^{i\phi_A}, \tag{21}$$

$$B = (|\langle B\rangle| + \delta B)e^{i\phi_B}, \tag{22}$$

$$C = -i(|\langle C\rangle| + \delta C)e^{i\phi_C}, \tag{23}$$

where δA, δB, and δC describe amplitude fluctuations, and ϕ_A, ϕ_B, and ϕ_C describe phase fluctuations of the fields.

Keeping linear fluctuation terms only we derive an equation for $\phi_A - \phi_B - \phi_C$ from (9) and (10). This equation shows that the evolution of the phase difference is stable, hence

$$\dot{\phi}_A - \dot{\phi}_B - \dot{\phi}_C = 0. \tag{24}$$

On the other hand

$$\frac{\dot{\phi}_B}{\gamma} - \frac{\dot{\phi}_C}{\gamma_M} = \frac{1}{\gamma}\left[\frac{\omega_- - \omega_b}{\gamma}\frac{F_B + F_B^\dagger}{2|\langle B\rangle|} + \frac{F_B - F_B^\dagger}{2i|\langle B\rangle|}\right] - \frac{1}{\gamma_M}\left[\frac{\omega_M - \omega_c}{\gamma_M}\frac{F_C - F_C^\dagger}{2i|\langle C\rangle|} + \frac{F_C + F_C^\dagger}{2|\langle C\rangle|}\right]. \tag{25}$$

Introducing phase diffusion coefficient as $\langle\phi^2\rangle - (\langle\phi\rangle)^2 = 2D\ t$, and taking in mind that the output power of the signal and idler can be written as $W_{-\ out} = \hbar\omega_-^2|\langle B\rangle|^2/Q$ and $W_{M\ out} = \hbar\omega_M^2|\langle C\rangle|^2/Q_M$ we derive from (24) and 25)

$$D_B = \frac{\gamma^2}{(\gamma+\gamma_M)^2}D_A + \frac{\gamma^2\gamma_M^2}{(\gamma+\gamma_M)^2}\frac{\hbar\omega_-}{W_{-\ out}}\left(1 + \frac{(\omega_- - \omega_b)^2}{\gamma^2}\right) \tag{26}$$

$$D_C = \frac{\gamma_M^2}{(\gamma+\gamma_M)^2}D_A + \frac{\gamma^2\gamma_M^2}{(\gamma+\gamma_M)^2}\frac{\hbar\omega_M}{W_{M\ out}}\left(1 + \frac{(\omega_M - \omega_c)^2}{\gamma_M^2}\right), \tag{27}$$

where D_A is the diffusion coefficient for the pump field. This coefficient is determined by the source of the pump. Because the quality factor of the whispering gallery modes may be very high we are able to get a stable generation in our system.

III. UP- AND DOWN-CONVERSION OF LIGHT INTO LIGHT AND MICROWAVES: ARTIFICIAL $\chi^{(3)}$ NONLINEARITY

Considered above parametric interaction couples two light modes and a single microwave mode. The microwave field has a frequency nearly resonant with the frequency difference of the pump and signal light. This picture is valid only if the optical modes are not equidistant, otherwise the pump light interacts with two optical modes having frequencies $\omega_{b\pm} \simeq \omega_a \pm \omega_c$. The condition for parametric oscillations drastically changes in this case.

The Hamiltonian describing this system is

$$\hat{H} = \hat{H}_0 + \hat{V}. \tag{28}$$

$\hat{H}_0$ is the free part of the Hamiltonian

$$\hat{H}_0 = \hbar\omega_a\hat{a}^\dagger\hat{a} + \hbar\omega_{b-}\hat{b}_-^\dagger\hat{b}_- + \hbar\omega_{b+}\hat{b}_+^\dagger\hat{b}_+ + \hbar\omega_c\hat{c}^\dagger\hat{c}, \tag{29}$$

where ω_a and $\omega_{b\pm}$ are the eigenfrequencies of the optical cavity modes, ω_c is the eigenfrequency of the microwave cavity mode, $\hat{a}$, $\hat{b}_\pm$, and $\hat{c}$ are the annihilation operators for these modes respectively.

The interaction part of the Hamiltonian is

$$\hat{V} = \hbar g(\hat{b}_-^\dagger\hat{c}^\dagger\hat{a} + \hat{b}_+^\dagger\hat{c}\hat{a}) + \text{adjoint}. \tag{30}$$

Instead of equations Eqs. (8-10) in this case we write

$$\dot{A} = -\Gamma_A A - ig(B_- C + C^\dagger B_+) + F_A, \tag{31}$$

$$\dot{B}_- = -\Gamma_{B-}B_- - igC^\dagger A + F_{B-}, \tag{32}$$

$$\dot{B}_+ = -\Gamma_{B+}B_+ - igCA + F_{B+}, \tag{33}$$

$$\dot{C} = -\Gamma_C C - ig(B_-^\dagger A + A^\dagger B_+) + F_M, \tag{34}$$

where

$$\Gamma_A = i(\omega_a - \omega_0) + \gamma,$$
$$\Gamma_{B\mp} = i(\omega_{b\mp} - \omega_\mp) + \gamma,$$
$$\Gamma_C = i(\omega_c - \omega_M) + \gamma_M,$$

A, $B_\pm$, and C are the slowly-varying amplitudes of the cavity mode operators; the optical (γ) and microwave (γ_M) decay rates as well as pumping forces F_A, $F_{B\pm}$, and F_M are introduced similarly to (11) and (12).

Let us solve set (31-34) in the steady state keeping expectation values only. From (32) and (33) we get

$$\langle B_-\rangle = -i\frac{g\langle C^*\rangle\langle A\rangle}{\Gamma_{B-}}\ ,\quad \langle B_+\rangle = -i\frac{g\langle C\rangle\langle A\rangle}{\Gamma_{B+}}\ . \tag{35}$$

Substituting (35) into (34) we derive (c.f. Eq. (15))

$$\langle\dot{C}\rangle + \left(\Gamma_C - g^2|\langle A\rangle|^2\frac{\Gamma_{B+} - \Gamma_{B-}^*}{\Gamma_{B+}\Gamma_{B-}^*}\right)C = 0. \tag{36}$$

This equation has a nontrivial steady state solution if the expression in parentheses is equal to zero (c.f. Eqs. (16) and (17))

$$\gamma_M = \frac{g^2|\langle A\rangle|^2\gamma[(\omega_{b+} - \omega_+)^2 - (\omega_{b-} - \omega_-)^2]}{[\gamma^2 + (\omega_{b+} - \omega_+)^2][\gamma^2 + (\omega_{b-} - \omega_-)^2]}, \tag{37}$$

$$\omega_c - \omega_M = \frac{\gamma_M}{\gamma}\frac{(\omega_{b+} - \omega_+)(\omega_{b-} - \omega_-) - \gamma^2}{(\omega_{b+} - \omega_+) + (\omega_{b-} - \omega_-)}. \tag{38}$$

Equation (37) determines threshold of the parametric oscillations. It shows the power of the pumping light that sustains generation of microwave field along with light sidebands. It is easy to see that no oscillations occur for any $\langle A\rangle$ if $(\omega_{b-} - \omega_-)^2 \geq (\omega_{b+} - \omega_+)^2$. On the other hand, if $(\omega_{b+} - \omega_+)^2 \gg (\omega_{b-} - \omega_-)^2$ we return to the case of usual parametric oscillator considered in the previous section. Threshold pump power can be written as

$$\widetilde{W}_{th} = W_{th}\frac{[\gamma^2 + (\omega_{b+} - \omega_+)^2][\gamma^2 + (\omega_{b-} - \omega_-)^2]}{\gamma^2[(\omega_{b+} - \omega_+)^2 - (\omega_{b-} - \omega_-)^2]}, \tag{39}$$

where we put $\Gamma_a = \gamma$, W_{th} is determined in (18).

Oscillation frequencies can be found from (38) if we take into attention that

$$\omega_{\pm} = \omega_0 \pm \omega_M. \tag{40}$$

Analysis of (38) shows that the frequency of the microwaves is determined either by ω_c, if $\gamma \gg \gamma_M$, or by $\omega_{b+} - \omega_{b-}$, if $\gamma_M \gg \gamma$. There is also a ratio between signal and idler amplitudes similar to (20)

$$\frac{|\langle B_- \rangle|^2 - |\langle B_+ \rangle|^2}{|\langle C \rangle|^2} = \frac{\gamma_M}{\gamma} \tag{41}$$

Let us consider, for instance, a nonlinear cavity with exactly equidistant spectrum

$$\omega_{b\pm} = \omega_a \pm \widetilde{\omega}_c, \tag{42}$$

where $\widetilde{\omega}_c$ is the frequency difference between the modes. Frequency of the microwave field ω_c is not necessary equal to $\widetilde{\omega}_c$. Then $\widetilde{W}_{th}$ (18) is inversely proportional to product $(\omega_a - \omega_0)(\widetilde{\omega}_c - \omega_M)$. Therefore, there is no parametric process for any pump pump power if the pump is resonant with a cavity mode $\widetilde{W}_{th} \to \infty$ if $|\omega_a - \omega_0| \to 0$. However, as is shown in the previous section, usual three-mode parametric process is the most efficient for the case of resonant pump tuning.

Let us consider now the case when $\gamma \gg \gamma_M$, $(\omega_{b+} - \omega_+)^2 \gg (\omega_{b-} - \omega_-)^2$ $(|\langle B_- \rangle|^2 \gg |\langle B_+ \rangle|^2)$ and find the phase diffusion coefficient for beat note for modes B_+ and B_-. To do it we introduce amplitude and phase fluctuation similar to (21-23)

$$A = (|\langle A \rangle| + \delta A) e^{i(\phi_A + \varphi_A)}, \tag{43}$$

$$B_{\pm} = (|\langle B_{\pm} \rangle| + \delta B_{\pm}) e^{i(\phi_{B\pm} + \varphi_{B\pm})}, \tag{44}$$

$$C = (|\langle C \rangle| + \delta C) e^{i(\phi_C + \varphi_C)}, \tag{45}$$

where $\varphi_\xi = \langle \xi \rangle / |\langle \xi \rangle|$ is the expectation value of the field phase.

Using this approximation we derive following expressions that connect phases of the pump and generated fields (c.f. [3])

$$\dot{\phi}_A = \dot{\phi}_{B-} + \dot{\phi}_C = \dot{\phi}_{B+} - \dot{\phi}_C, \tag{46}$$

$$\dot{\phi}_C \simeq \frac{\omega_M - \omega_c}{\gamma_M} \frac{F_C - F_C^\dagger}{2i|\langle C \rangle|} + \frac{F_C + F_C^\dagger}{2|\langle C \rangle|}.$$

Therefore, the phase of the signals beat note is determined by phase diffusion of the microwave field $\phi_{B+} - \phi_{B-} = 2\phi_C$ has a diffusion coefficient

$$D \simeq 4\gamma_M^2 \frac{\hbar \omega_M}{W_{M\ out}} \left(1 + \frac{(\omega_M - \omega_c)^2}{\gamma_M^2} \right). \tag{47}$$

Such parametric oscillations have much in common with near resonant four wave mixing produced in atomic vapors [23]. Namely, i) in those experiments Stokes and anti-Stokes optical fields where generated spontaneously from vacuum; the same is expected in our case; ii) the frequency difference between anti-Stokes and pump fields and pump and Stokes fields was equal to the hyperfine splitting of the ground state of rubidium atoms; the frequency difference is determined by the frequency of the microwave field in our case; iii) threshold of the oscillations in atomic medium was a few μW for the pump power; the same level of the threshold pump power is expected in our case; iv) in atomic experiments the oscillations become possible due to long lived atomic coherence; in our case the role of atomic coherence is played by the microwave mode; v) phase diffusion of the beat note of the Stokes and anti-Stokes fields generated in atomic system is determined by the atomic coherence life time [37]; in our case it is determined by the quality factor of the microwave mode (47). Therefore, we have constructed an artificial resonant $\chi^{(3)}$ nonlinearity using nonresonant $\chi^{(2)}$ nonlinear medium.

There is a difference, however, between resonant four-wave mixing in atomic medium and in our system: i) the atomic medium has essentially nonlinear response that leads, in particular, to creation of a "dark state" that does not interact with multifrequency light [38]; there is no such a state in our nonlinear system; ii) the Stokes and anti-Stokes fields have nearly the same amplitudes in atomic medium. In our case the fields have different amplitudes.

IV. OPTICAL COMB GENERATION

Optical comb generation can be achieved using electro-optical modulator with external microwave pumping [9, 10]. Resonant atomic and molecular systems may lead to efficient generation of a comb of optical frequencies without such pumping [24, 25]. It is known also that whispering gallery modes result in enhancement of Raman scattering [26–29]. Let us study a possibility of a comb generation in our resonant parametric system.

Let us assume that our system consisting of nonlinear oblate spheroid microcavity and microwave resonator is pumped by a two-frequency light. Each mode of the pumping is resonant with a mode of the cavity. The frequency difference for the pumping is equal to the resonant frequency of the microwave resonator. We assume also that the cavity modes are equidistant and frequency difference between them is equal to the microwave frequency. This is true in the first approximation for the spheroid [30]. However, even the residual dispersion can be compensated [31].

Under such conditions two optical fields generate microwave field in our system. The microwave field interacts with the light and generate equidistant frequency spectrum. This process is similar rather to the comb generation in atomic medium [24, 25] than to the usual comb generation technique, where microwave field, applied to a nonlinear crystal, modulates light [9, 10].

To describe the comb generation we write interaction

Hamiltonian as

$$\hat{V} = \hbar g \sum_{n=-\infty}^{\infty} (\hat{a}_{n-1}^{\dagger}\hat{c}^{\dagger}\hat{a}_n + \hat{a}_{n+1}^{\dagger}\hat{c}\hat{a}_n) + \text{adjoint}. \quad (48)$$

where $\hat{a}_n$ is annihilation operator for n$^{\text{th}}$ cavity mode. We assume that modes are completely identical with respect to their quality factor and coupling to the microwave field.

Using (48) we derive equations of motion for the modes. For the sake of simplicity we consider the case of exact resonance for all the modes. In slowly varying amplitude and phase approximation equations for the expectation values of the field amplitudes are

$$\dot{A}_n = -\gamma A_n - ig(A_{n-1}C + C^* A_{n+1}) + F_n(\delta_{n,0} + \delta_{n,-1}) \quad (49)$$

$$\dot{C} = -\gamma_M C - ig \sum_{n=-\infty}^{\infty} (A_{n-1}^* A_n + A_n^* A_{n+1}), \quad (50)$$

where F_n stands for the pumping of the modes, $\delta_{i,j} = 1$ if $i = j$ and $\delta_{i,j} = 0$ if $i \neq j$. In other words, we assume that only modes with $n = 0$ and $n = -1$ are pumped.

We introduce function

$$A(\theta) = \sum_{n=-\infty}^{\infty} A_n e^{i\theta n}, \quad (51)$$

and present the amplitude of the microwave field as $C = |C| \exp(i\phi_C)$. Then rewriting Eqs. (49) in steady state

$$A_n = -i\frac{g}{\gamma}(A_{n-1}C + C^* A_{n+1}) + \frac{F_n}{\gamma}(\delta_{n,0} + \delta_{n,-1}), \quad (52)$$

multiplying each of them on $\exp(i\theta n)$ (n corresponds to the index of term γA_n), and summarize them over all n we derive

$$A(\theta) = \frac{F_0 + F_{-1}e^{-i\theta}}{\gamma + 2ig|C|\cos(\theta + \phi_C)}. \quad (53)$$

The solution for each mode A_n can be written as

$$A_n = \frac{1}{2\pi} \int_0^{2\pi} A(\theta) e^{-i\theta n} d\theta. \quad (54)$$

Eq. (53) contains unknown constants $|C|$ and ϕ_C. To find them we write Eq. (50) in the steady state

$$C = -i\frac{g}{\gamma_M} \sum_{n=-\infty}^{\infty} (A_{n-1}^* A_n + A_n^* A_{n+1}), \quad (55)$$

and substitute there A_n (52). This gives us $\phi_C = -\pi/2 +$ arg $(F_{-1}^* F_0)$ and equation for $|C|$

$$|C| = \frac{g}{\gamma\gamma_M} \sum_{n=-\infty}^{\infty} |A_{n-1}^* F_n + F_n^* A_{n+1}|(\delta_{n,0} + \delta_{n,-1}) =$$
$$\frac{2g}{\pi\gamma_M}|F_{-1}^*||F_0| \int_0^{2\pi} \frac{\sin^2(\theta + \phi_C) d\theta}{\gamma^2 + 4g^2|C|^2\cos^2(\theta + \phi_C)} =$$
$$\frac{4g}{\gamma_M}\frac{|F_{-1}^*||F_0|}{4g^2|C|^2}\left(\sqrt{1 + \frac{4g^2|C|^2}{\gamma^2}} - 1\right). \quad (56)$$

Solution of this equation gives us the amplitude of the microwave field.

Finally we note that actual time dependent amplitude of the light can be written as

$$A(t) = e^{-i\omega_0 t} \sum_{n=-\infty}^{\infty} A_n e^{-i\omega_M n t}, \quad (57)$$

where ω_0 is the carrier frequency of the mode with $n = 0$. Exchanging θ with $-\omega_M t$ in (53) and (54) we derive

$$A(t) = \frac{F_0 e^{-i\omega_0 t} + F_{-1}e^{-i(\omega_0 - \omega_M)t}}{\gamma + 2ig|C|\cos(\omega_M t - \phi_C)}. \quad (58)$$

Let us note here that the signal generated in our system is different from the usual phase modulated signal. This occurs because of the saturation of the oscillations.

The width of the frequency comb is determined by value $g|C|/\gamma$. To have a wide spectrum this value should be comparable with a unity. Assuming that both pump harmonics have the same power W_A and, therefore, $|F_{-1}| \approx |F_0|$ we get

$$\frac{g^2}{\gamma\gamma_M}\frac{|F_{-1}||F_0|}{\gamma^2} = \frac{W_A}{W_{th}} \geq 1,$$

where W_{th} is the threshold power for the parametric oscillations (18). As we has discussed above this power can be as low as μW for realistic conditions. Therefore it is possible to generate a broad frequency comb in our system using small pump power.

V. MULTIMODE REGIME OF PONDEROMOTIVE PARAMETRIC INSTABILITY

It was shown recently that long-base gravitational wave detectors may safer from parametric instability [21]. This instability arises from ponderomotively mediated coupling between mechanical oscillations of suspended cavity mirrors and probe light that is used for detection of the mirrors' signal shift. The effect is undesirable because it might create a specific upper limit for the energy stored in the cavity. The sensitivity of the detection increases with the light power and, hence, such a process might pose an upper limit on the measurement sensitivity.

Planed circulating power in the interferometer is about 1 MW. It was shown that this power exceeds the threshold for parametric oscillations almost 300 times for realistic conditions if the optical mode spectrum is not equidistant (three-wave description of the parametric process is valid) [21].

We can easily describe the ponderomotive parametric instability using the technique presented above. Really, the interaction between mirrors' mechanical oscillations and optical modes can be characterized by either Eqs. (8-9) or Eqs. (31-34), where C describes mechanical oscillations and the coupling factor g is appropriately chosen.

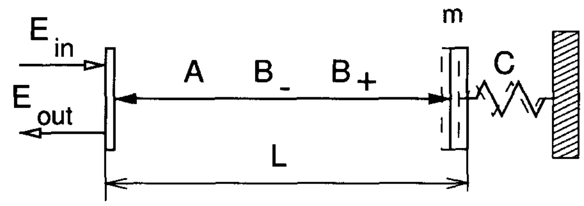

FIG. 2: Fabry-Perot cavity with movable mirror. Parametric instability is possible due to ponderomotive nonlinearity.

Let us consider a Fabry-Perot interferometer with one movable mirror that has mass m and mechanical resonance frequency ω_c (Fig. 2). Distance between mirrors of the resonator is equal to L. Then the coupling constant between mechanical degrees of freedom of the mirror and optical modes is

$$g = \frac{\omega_a}{L}\sqrt{\frac{\hbar}{2m\omega_c}}. \tag{59}$$

In the case of not equidistant mode spectrum we may consider only two optical modes and mechanical mode and use Eqs. (8-9) that gives us the result of [21]. Threshold power for the oscillations for for resonant tuning of the pumping laser follows from

$$\frac{2W_{th}}{\gamma}\frac{2QQ_M}{m\omega_M^2 L^2} = 1. \tag{60}$$

In the case of nearly equidistant modes we should use Eqs. (31-34). Then the threshold power increases according to (39). Because for long base interferometers the main longitudinal mode spectrum is almost equidistant we might expect that the threshold of the parametric oscillations will increase significantly. The problem may arise, however, due to transverse modes of the system. Therefore, to understand if the system is stable one needs to consider exact mode structure of a particular system.

VI. CONCLUSION

We have shown that a strongly nondegenerate multifrequency parametric oscillator possesses by different properties compared with usual three-wave OPO. As examples of such an oscillator, we have studied a scheme of all-resonant optical-microwave parametric oscillator based on whispering gallery modes excited in a nonlinear dielectric optical cavity and a long-base cavity with moving walls.

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